The Bose-Einstein Condensate (BEC) problem:

The linear Hamiltonian is the harmonic oscillator Hamiltonian:

H0 = P^2/2 + X^2/2

when P is the momentum operator (-i\*d/dx), and P^2 is equivalent to:

-d^2/dx^2

The non-linear Hamiltonian is:

Vnl = conj(u)\*u

when u is the state vector.

The initial state is the ground state, given an initial momentum:

exp(1i\*x)\*(ground state vector)

The ground state is given by an iterative process.

The RK4 data:

time steps matvecs Relative L2 Error

1.5000000e+03 6.0000000e+03 2.1908936e-08

1.8880000e+03 7.5520000e+03 8.7288844e-09

2.3770000e+03 9.5080000e+03 3.4740341e-09

2.9930000e+03 1.1972000e+04 1.3820096e-09

3.7680000e+03 1.5072000e+04 5.5015404e-10

4.7430000e+03 1.8972000e+04 2.1913306e-10

5.9720000e+03 2.3888000e+04 8.7182987e-11

7.5180000e+03 3.0072000e+04 3.4713134e-11

9.4640000e+03 3.7856000e+04 1.3823129e-11

1.1915000e+04 4.7660000e+04 5.5023498e-12

1.5000000e+04 6.0000000e+04 2.1932741e-12

1.8884000e+04 7.5536000e+04 8.7836428e-13

The new algorithm data:

for m = 5, k = 12:

time steps matvecs Relative L2 Error

2.0000000e+02 3.2320000e+03 6.1764200e-10

2.5200000e+02 4.0640000e+03 3.0220785e-11

3.1700000e+02 5.1040000e+03 4.9093645e-12

3.9900000e+02 6.4160000e+03 1.1782762e-12

for m = 5, k = 9:

time steps matvecs Relative L2 Error

3.0000000e+02 3.9260000e+03 2.8271685e-10

3.7800000e+02 4.9400000e+03 1.4693141e-11

4.7500000e+02 6.2010000e+03 9.9325708e-13

5.9900000e+02 7.8130000e+03 1.6526831e-13

for m = 7, k = 12:

time steps matvecs Relative L2 Error

1.9000000e+02 3.4560000e+03 1.1526037e-04

1.9900000e+02 3.6180000e+03 7.6453864e-12

2.0800000e+02 3.7800000e+03 4.5629806e-13